Pset 1 9/8/2024

King

1.2 $\omega = 2\pi\nu = 880\pi\ s^{-1}$

$\therefore x(t) = 5\times10^{-4}[m]\cos(880\pi t)$ ← use $x(t)$ to find $v(t)$, $a(t)$

$x = A$ at $t=0$, $x=0$ at $\cos(880\pi t) = 0$

$$880\cancel{\pi}t = \cancel{\pi}/2$$

$$t = \frac{1}{1760}\ s$$

$v(t) = -5\times10^{-4}\cdot 880\pi \sin(880\pi t)$

max v at $x=0$, $t = \frac{1}{1760}$ s) plug in

$\therefore |v_{max}| = 5\times10^{-4}\cdot 880\pi \cdot \cancel{\sin\left(\frac{\pi}{2}\right)}^{1}$

$= 0.44\pi\ ms^{-1} = \boxed{1.38\ m\,s^{-1}}$

$a(t) = -5\times10^{-4}\cdot(880\pi)^2\cos(880\pi t)$

max a at turning points, e.g. $t=0$.

$\therefore |a_{max}| = 5\times10^{-4}\cdot(880\pi)^2$

$= 387.2\pi^2\ m\,s^{-2} = \boxed{3.82\times10^3\ ms^{-2}}$

1.3 if platform accelerates downward too fast, $|a| > 9.81\ ms^{-2}$, then anything resting on it will lose contact. $\therefore$ I need to find max ν s.t. $|a| \le 9.81\ ms^{-2}$.

$a(t) = -0.2[m]\cdot(2\pi\nu)^2\cos(2\pi\nu t)$.

$x = A$ at $t=0$. $\therefore$ max $|a|$ also at $t=0$

$|a(t)| = 0.2\cdot4\pi^2\cdot\nu^2\cdot 1 \le 9.81$

$$\nu^2 \le \frac{9.81}{0.8\pi^2} = 1.242451\ s^{-2}$$

$\therefore \boxed{\nu_{max} = 1.11\ Hz}$

1.5 $k = 400\ kg\,s^{-2}$, $m = 0.75\,kg$ $\Rightarrow$ $\omega = \sqrt{\frac{k}{m}} = \underline{23.09\ s^{-1}}$

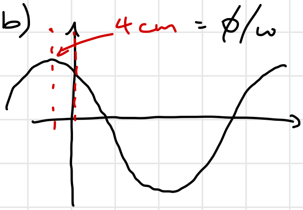
a) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 0.2721\,s$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\cdot 0.75\cdot 0.5^2 = 0.09375\,J$$
$$U = \int_0^{0.04[m]} kx\,dx = \frac{1}{2}kx^2\Big|_0^{0.04} = \frac{1}{2}\cdot 400\cdot 0.0016 = 0.32\,J$$
} self explanatory

Since $E = K + U$, we can sum the above values to get total energy!

$E = 0.41375\,J \simeq \boxed{0.41\,J}$

b) 4 cm $= \phi/\omega$



$$|x(t)| = A\cos(\cancel{\omega t}^{\,0} + \phi) = A\cos\phi = 0.04\,m$$
$$|v(t)| = \omega A\sin(\cancel{\omega t}^{\,0} + \phi) = \omega A\sin\phi = 0.50\,ms^{-1}$$

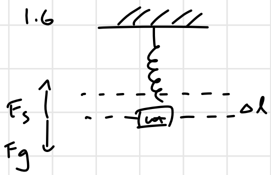
manipulate to get $\cos^2\phi + \sin^2\phi$

$$A\cos\phi = 0.04\,m$$
$$A\sin\phi = 0.50\,ms^{-1} / 23.09\,s^{-1} = 0.02165\,m$$
$$A^2(\cancel{\cos^2\phi + \sin^2\phi})^{1} = 0.00206872\,m^2 \Rightarrow A = \underline{0.04548\,m}$$

$$x(t) = 0.04548\cos\phi = -0.04$$
$$\Rightarrow \cos\phi = -0.8795$$
$$\phi = \arccos(-0.8795) = \underline{2.65\ rad}$$

$$\boxed{x(t) = 0.045\cos(23.1t + 2.7)}$$

1.6

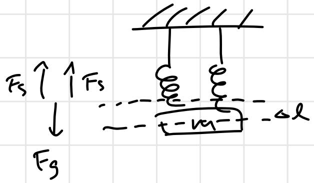


from class notes

$$mg = k\Delta l$$

$$F = ma = m\frac{d^2x}{dt^2} = mg - k(\Delta l + x)$$
$$= \cancel{mg - k\Delta l}^{\to 0} - kx$$

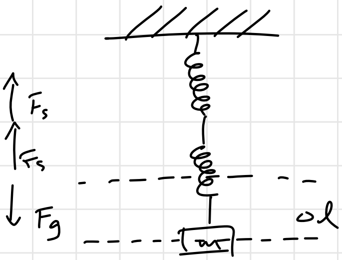
$$\Rightarrow \omega^2 = k/m$$



$mg = 2k\Delta l$ because 2 springs are pulling it up.

$$F = ma = m\frac{d^2x}{dt^2} = mg - 2k(\Delta l + x)$$
$$= \cancel{mg - 2k\Delta l}^{\to 0} - 2kx \quad \text{similar to above}$$

$$\frac{d^2x}{dt^2} = \frac{-2k}{m}x \rightarrow \omega^2 = 2k/m$$



in series, both springs stretch. Tension is equal in " " .

$$F = k_1 x_1, \quad F = k_2 x_2.$$
$$k_1 = k_2, \quad x_1 = x_2$$
$$\Rightarrow \Delta l = x_1 + x_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\Delta l = \frac{2F}{k} \Rightarrow \Sigma \vec{F}_s = \frac{k\Delta l}{2} \quad \text{total spring force}$$

from above: $mg = \frac{k\Delta l}{2}$

$$F = ma = m\frac{d^2x}{dt^2} = mg - \frac{k}{2}(\Delta l + x)$$
$$= \cancel{mg - \frac{k\Delta l}{2}}^{\to 0} - \frac{kx}{2} \Rightarrow \omega^2 = k/2m$$

a: $\omega = \sqrt{2}\sqrt{\frac{k}{m}}$

b: $\omega = \sqrt{\frac{k}{m}}$

c: $\omega = \frac{1}{\sqrt{2}}\sqrt{\frac{k}{m}}$

$\therefore$ ratio is indeed $\sqrt{2} : 1 : \sqrt{\frac{1}{2}}$

1.11 a) $F = -\nabla U = -\frac{dU}{dx} = \frac{d}{dx}\left(\frac{a}{x^6} - \frac{b}{x^{12}}\right)$

$$= \frac{-6a}{x^7} + \frac{12b}{x^{13}} = 0 \text{ for equilibrium}$$

$$\frac{6a}{x^7} = \frac{12b}{x^{13}}$$

$$x^6 = \frac{2b}{a} \Rightarrow \boxed{x_0 = \left(\frac{2b}{a}\right)^{1/6}}$$

b) small disp. = Taylor series.

from class notes: use 2nd derivative term

$$U \simeq \frac{(x-x_0)^2}{2} \underbrace{\left(\frac{d^2U}{dx^2}\right)_{x=x_0}}_{k}$$

plug in part a)

$$\frac{d^2U}{dx^2} = \frac{42a}{x^8} - \frac{156b}{x^{14}} \Rightarrow \frac{42a}{\left(\frac{2b}{a}\right)^{4/3}} - \frac{156b}{\left(\frac{2b}{a}\right)^{7/3}}$$

$$= \frac{21a^2}{b} \cdot \left(\frac{a}{2b}\right)^{1/3} - \frac{39a^2}{b}\left(\frac{a}{2b}\right)^{1/3}$$

$$= -18a \cdot \frac{a}{b} \cdot \left(\frac{a}{2b}\right)^{1/3}$$

$$= -36a \cdot \frac{a}{2b}\left(\frac{a}{2b}\right)^{1/3}$$

$$= -36a\left(\frac{a}{2b}\right)^{4/3}$$

$$= -k$$

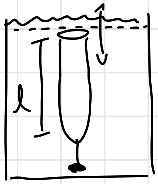
$$F = m\ddot{x} = -kx$$

$$\Rightarrow \ddot{x} = -\frac{k}{m}x, \text{ where } \boxed{k = 36a\left(\frac{a}{2b}\right)^{4/3} \text{ and } \omega^2 = \frac{k}{m}}$$

Pset 2 9/18/2024

1.7

a)



$F_b = F_g$

$[F] = kg\,m\,s^{-2}$

$[A] = m^2$

$[\rho] = kg\,m^{-3}$

$[g] = m\,s^{-2}$

use dimensional analysis to get equation for buoyancy force.

$[A\rho g] = kg\,s^{-2}$

$\Rightarrow [A\rho g l] = kg\,m\,s^{-2}$ l is part in water.

Hence, $F_b - F_g = mg - A\rho g l = 0$

$$m\ddot{x} = mg - A\rho g(l+x) = \cancel{mg - A\rho g l}^{\,0} - A\rho g x$$

$$m\ddot{x} = -A\rho g x \Rightarrow \ddot{x} = -\frac{A\rho g}{m}x, \quad \omega = \sqrt{\frac{A\rho g}{m}}$$

b) $m\ddot{x} = m\frac{dv}{dt} = \frac{-A\rho g}{m}x$ multiply both sides by $vdt = dx$

$\Rightarrow mvdv = -A\rho g x\, dx$

$d\left(\frac{1}{2}mv^2\right) = d\left(-\frac{1}{2}A\rho g x^2\right)$ antiderivatives.

$\underbrace{\frac{1}{2}mv^2}_{KE} + \underbrace{\frac{1}{2}A\rho g x^2}_{PE} = \text{const.}$ $\rightarrow$ Total energy $= \frac{1}{2}mv^2 + \frac{1}{2}A\rho g x^2$

PE $= \frac{1}{2}A\rho g x^2$

1.8 $[s] \propto [kg]^{\alpha}[m]^{\beta}[m s^{-2}]^{\gamma}$

immediately, we see that there is nothing to cancel out kg, so it seems mass is not used. from the remaining values, if you cancel out masses, all you need is to multiply s^{-2} by $s^{-1/2}$, i.e. $\gamma = -1/2$. $\Rightarrow \beta = 1/2$

$$\alpha = 0, \beta = 1/2, \gamma = -1/2, \quad T \propto l^{1/2} \cdot g^{-1/2} = \sqrt{\frac{l}{g}}$$

1.10 $\tau = I\frac{d^2\theta}{dt^2}$, $I = \frac{ML^2}{3}$

$$\tau = r \times F = L \times -kL\sin\theta \ (\simeq \theta)$$
$$= -kL^2\theta$$

$$\frac{M\cancel{L^2}}{3}\frac{d^2\theta}{dt^2} = -k\cancel{L^2}\theta \rightarrow \frac{d^2\theta}{dt^2} = -\frac{3k}{M}\theta \Rightarrow \omega = \sqrt{\frac{3k}{M}}$$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{3k}}$$

1.12 KE of spring: $dm = \frac{m}{L}\,dl$, $\frac{m}{L}$ is mass/length so integrating gives mass.

a) $dK = \frac{1}{2}\,dm\,v(l)^2$, self explanatory

$v(l) = \frac{l}{L} v_{end}$, parts of spring have diff. v.

$$\Rightarrow \int dK = \int_0^L \frac{1}{2}\frac{m}{L}\left(\frac{l}{L}v_{end}\right)^2 dl = \frac{1}{2}\frac{m}{L^3}v_{end}^2 \int_0^L l^2\,dl$$

$$= \frac{1}{2}\frac{m}{\cancel{L^3}}v_{end}^2 \frac{\cancel{L^3}}{3}$$

$$= \frac{1}{2}\frac{m}{3}v_{end}^2$$, where v_{end} = v of mass M.

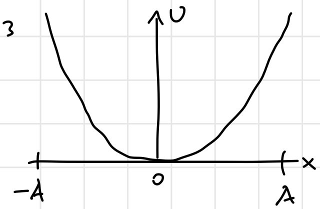
Hence $E = K + U = \frac{1}{2}(M + m/3)v^2 + \frac{1}{2}kx^2$

b) $\frac{dE}{dt} = (M + m/3)\,v\frac{dv}{dt} + kx\frac{dx}{dt} = 0$ $\quad P = \frac{dE}{dt} = Fv$ and we want F.

$$(M + m/3)\cancel{v}\,\ddot{x} = -kx\cancel{v}$$

$$\Rightarrow \ddot{x} = -\frac{k}{M + m/3}x, \quad \omega = \sqrt{\frac{k}{M + m/3}}$$

1.13



$U(x) = \frac{1}{2}kx^2$. $U(A) = \frac{1}{2}kA^2$.

$K = \frac{1}{2}k(A^2 - x^2)$ ← from lecture.

$\therefore K = U(A) - U(x)$

$\frac{1}{2}mv^2 = U(A) - U(x)$

$$\Rightarrow v = \sqrt{\frac{2[U(A) - U(x)]}{m}}$$

b) $dx = v(x)\,dt \Rightarrow dt = \frac{dx}{v(x)}$

$$T = \int dt = 4\int_0^A \frac{dx}{\sqrt{\frac{2}{m}[U(A) - U(x)]}} = 4\int_0^A \frac{dx}{\sqrt{\frac{\cancel{2}}{m}[\frac{1}{\cancel{2}}k(A^2 - x^2)]}} = 4\int_0^A \frac{dx}{\sqrt{\frac{k}{m}(A^2 - x^2)}}$$

integrate from $0 \to A$ & multiply by 2 bc of symmetry. Then multiply by 2 again for a full oscillation.

$$= 4\sqrt{\frac{m}{k}}\int_0^A \frac{dx}{\sqrt{A^2 - x^2}}$$

$k = \frac{2U(A)}{A^2}$ from above.

$$= 4\sqrt{\frac{m}{2U(A)}}\cdot\cancel{A}\int_0^A \frac{dx}{\cancel{A}\sqrt{1 - x^2/A^2}}$$

$\frac{x^2}{A^2} = \frac{U(x)}{U(A)}$

$$\Rightarrow T = 4\sqrt{\frac{m}{2U(A)}}\int_0^A \frac{dx}{\sqrt{1 - \frac{U(x)}{U(A)}}}$$

c) $U(x) = \alpha x^n$, $\quad U(A) = \alpha A^n$

$$\Rightarrow \frac{U(x)}{U(A)} = \left(\frac{x}{A}\right)^n = \xi^n$$

$$\xi = \frac{x}{A} \qquad d\xi = \frac{dx}{A}$$

$$T = 4\sqrt{\frac{m}{2\alpha A^n}} \int_0^A \frac{dx}{\sqrt{1-\xi^n}} \qquad \leftarrow \; = A\,d\xi$$

x	ξ
0	0
A	1

$$= 4\sqrt{\frac{m}{2\alpha A^n}} \int_0^1 \frac{A\,d\xi}{\sqrt{1-\xi^n}}$$

$$= 4A \cdot A^{-n/2}\sqrt{\frac{m}{2\alpha}} \underbrace{\int_0^1 \frac{d\xi}{\sqrt{1-\xi^n}}}_{\text{indep. of } A.}$$

$$\therefore \; T \propto A^{1-n/2}$$

$n=2$: no dependence

$n=4$: $T \propto A^{-1}$

$n=6$: $T \propto A^{-2}$

$\vdots$

Pset 3

2.1 $\vec{F}_d = -bv$. Find critical damping condition.

$b = m\gamma$. $m = 2.5\,kg$ $A_{max} = 0.06\,m$

$$mg = |kx| \Rightarrow 2.5 \times 9.81 = k \cdot 0.06$$
$$\Rightarrow k = 408.75\ Nm^{-1}$$
$$\omega_0^2 = \frac{k}{m} = \frac{\gamma^2}{4} = \frac{b^2}{4m^2} \quad \text{(condition for critical damping)}$$

$$\Rightarrow 2\sqrt{mk} = b \quad \Rightarrow b = 2 \cdot \sqrt{2.5 \times 408.75}$$
$$= 63.9\ kg\,s^{-1}$$

2.6 $\omega^2 = \omega_0^2 - \frac{\gamma^2}{4}$. $Q \equiv \frac{\omega_0}{\gamma}$. very lightly damped $\Rightarrow \omega_0^2 \gg \frac{\gamma^2}{4}$

fractional change: $\frac{\gamma^2}{4} = \frac{\omega_0^2}{8Q^2}$

$$\therefore \omega^2 = \omega_0^2 - \frac{\omega_0^2}{8Q^2} = \omega_0^2\left(1 - \underbrace{\frac{1}{8Q^2}}_{\text{change}}\right)$$

2.7 assuming b in both are equal, $\therefore$

Aluminum:
$$A_n = A_0 e^{-\gamma t/2} \Rightarrow \ln\left(\tfrac{1}{2}\right) = -bt/2m$$

Brass:
$$A_n = A_0 e^{-\gamma t/2} \Rightarrow \ln\left(\frac{A_n}{A_0}\right) = -bt/2m$$

$$\text{ratio: } \frac{\ln\left(\frac{1}{2}\right)}{\ln\left(\frac{A_n}{A_0}\right)} = \frac{\frac{1}{m_A}}{\frac{1}{m_B}}$$

$$\Rightarrow \ln\left(\frac{A_n}{A_0}\right) = \ln(0.5) \cdot \frac{M_A}{M_B} \Big\} \frac{\rho_A}{\rho_B} \text{ bc } V \text{ is same.}$$

$$\therefore \ln\left(\frac{A_n}{A_0}\right) = \ln(0.5)\frac{2.7}{8.5} = -0.2202$$

$$\Rightarrow A_n = A_0 \cdot e^{-0.2202} = 0.8\,A_0.$$

3.15

a) $\omega_0 = 2\pi f = 2\pi \cdot 256 = 512\pi$

(self-explanatory) $\frac{E}{E_0} = \frac{1}{2} = e^{-\gamma} \Rightarrow \gamma = \ln 2$ $\Rightarrow Q = \frac{\omega_0}{\gamma} = \frac{512\pi}{\ln 2} \approx 2320$

b) $\omega_0 = 1024\pi \Rightarrow Q \approx 4640$

c) $\tau = \frac{1}{\gamma} = 4 \Rightarrow \gamma = \frac{1}{4}$, $\gamma = \frac{b}{m} \Rightarrow b = m\gamma = \frac{1}{40}$

$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.9}{0.1}} = 3.$ $Q = \frac{\omega_0}{\gamma} = 12$

3.16 $\frac{dE}{dt} = -\frac{Ke^2a^2}{c^3}.$ t for 1 cycle is $\frac{2\pi}{2\pi\nu} = \frac{1}{\nu}$

a)

$x(t) = A\sin(2\pi\nu t)$

$\dot{x}(t) = 2\pi\nu A\cos(2\pi\nu t)$

$\ddot{x}(t) = -4\pi^2\nu^2 A\sin(2\pi\nu t)$

$$\Rightarrow \int_0^{1/\nu} -\frac{Ke^2a^2}{c^3}dt = \int_0^{1/\nu} -\frac{Ke^2 16\pi^4\nu^4A^2\sin^2(2\pi\nu t)}{c^3}dt = -\frac{16\pi^4Ke^2\nu^4A^2}{c^3}\int_0^{1/\nu}\frac{1-\cos(4\pi\nu t)}{2}dt$$

$$= -\frac{8\pi^4Ke^2\nu^4A^2}{c^3}\Big[t - \cancel{\frac{1}{4\pi\nu}\sin(4\pi\nu t)}^{\to 0}\Big]_0^{1/\nu}$$

$$\Rightarrow E \text{ Lost} = \frac{8\pi^4Ke^2\nu^3A^2}{c^3}$$

b) $\frac{E_1 - E_2}{E_1} = \frac{2\pi}{Q} \Rightarrow Q = \frac{2\pi E_1}{E_1 - E_2}$, $E_1 = \frac{1}{2}kx^2$ when $v = 0$. $k = m\omega_0^2 = m_e 4\pi^2\nu^2$

$\Rightarrow E_1 = \frac{1}{2}m_e 4\pi^2\nu^2 A^2 \underbrace{\sin(2\pi\nu t)}_{=1 \text{ when } v=0.}$

$$\Rightarrow Q = \frac{4\pi^3 m_e \nu^2 A^2 c^3}{8\pi^4 K e^2 \nu^3 A^2} = \frac{m_e c^3}{2\pi K \nu e^2}$$

c) $-\gamma t = \ln\frac{1}{2} \Rightarrow t = \frac{-\ln\frac{1}{2}}{\gamma} = \frac{Q}{\omega_0}\ln 2.$ $T = \frac{1}{\nu}$ $\therefore N = \frac{t}{T} = t\nu$ oscillations.

$$N = \frac{Q}{2\pi\cancel{\nu}}\cdot\ln 2\cdot\cancel{\nu} = \frac{m_e c^3}{4\pi^2 K \nu e^2}\ln 2$$

d) $\nu = 5\times10^{14}$ Hz (yellow)

plug values: $Q = 51$ million, $t = 1.12\times10^{-8}$ s coding: please read comments on my code. they explain what I am doing.

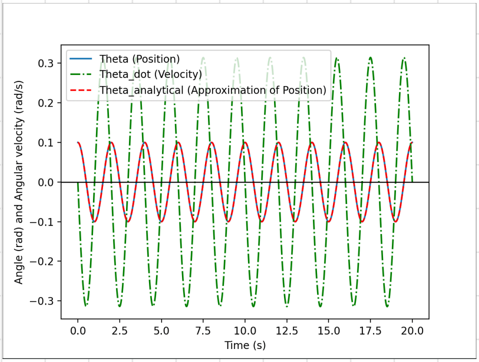
1.

b)

```python
import math
import numpy as np
import matplotlib.pyplot as plt

# initialize array starting at t=0 w/ 1000 steps, dtype=float
N = 1000
times = np.linspace(0, 20, N + 1)
timestep = 0.02

# assign constants
g = 9.8
l = g/(np.pi)**2

# initial conditions
theta_0 = 0.1    # value from pset
theta_dot_0 = 0    # value from pset

# lists which will be added on to with return values of RK2 function
thetas = [theta_0]
theta_dots = [theta_dot_0]

def RK2(timestep, theta, theta_dot):

    # Filling out everything according to formulas
    k1_theta = timestep * theta_dot
    k1_theta_dot = timestep * (-g/l)*np.sin(theta)

    k2_theta = timestep * (theta_dot + k1_theta_dot/2)
    k2_theta_dot = timestep * (-g/l)*np.sin(theta + k1_theta/2)

    # Update values
    theta_new = theta + k2_theta
    theta_dot_new = theta_dot + k2_theta_dot

    return theta_new, theta_dot_new

# assign values at t=0 to current
theta_current = theta_0
theta_dot_current = theta_dot_0

for i in range(N):
    # assigning return values to current values so the loop can continue
    theta_current, theta_dot_current = RK2(timestep, theta_current, theta_dot_current)
    # adding values to the list
    thetas.append(theta_current)
    theta_dots.append(theta_dot_current)

# compare to analytical solution (I am using what I see on 1d. I don't know why the Jupyter notebook has a more complicated version.)
omega = np.sqrt(g/l)
theta_analytical = theta_0 * np.cos(omega * times)


# table for pset 1c
print("Times, Thetas, Theta_dots:")
for i in range(11):
    print(f"Time: {times[i]}, Theta: {thetas[i]}, Theta_dot: {theta_dots[i]}")

# plotting
plt.figure(1)
plt.plot(times, thetas, label="Theta (Position)")
plt.plot(times, theta_dots, label="Theta_dot (Velocity)", linestyle="-.", color="green")
plt.plot(times, theta_analytical, label="Theta_analytical (Approximation of Position)", linestyle="--", color="red")
plt.axhline(y=0, color='black', linestyle='-', linewidth=1)
plt.xlabel("Time (s)")
plt.ylabel("Angle (rad) and Angular velocity (rad/s)")
plt.legend()

plt.figure(2)
plt.plot(times, (thetas-theta_analytical)**2)
plt.xlabel("time (s)", fontsize=20)
plt.ylabel(r"Residuals", fontsize=20)
plt.show()
```

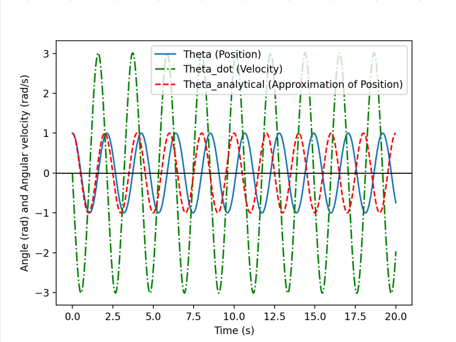
c)

i)



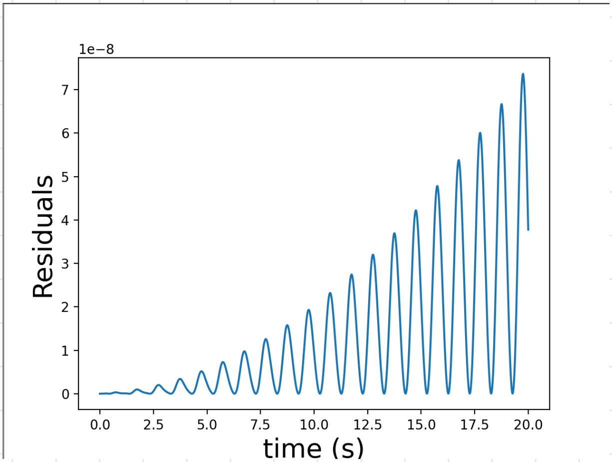

```
Times, Thetas, Theta_dots:
Time: 0.0, Theta: 0.1, Theta_dot: 0
Time: 0.02, Theta: 0.09980293673433734, Theta_dot: -0.019706326566266457
Time: 0.04, Theta: 0.09921213398515179, Theta_dot: -0.03933524280827243
Time: 0.06, Theta: 0.09822991334005482, Theta_dot: -0.058809557350423466
Time: 0.08, Theta: 0.09686013578980443, Theta_dot: -0.0780526685310158
Time: 0.1, Theta: 0.09510818699490983, Theta_dot: -0.09698885753469336
Time: 0.12, Theta: 0.09298095670986926, Theta_dot: -0.11554357907837533
Time: 0.14, Theta: 0.0904868124224158, Theta_dot: -0.1336437487919696
Time: 0.16, Theta: 0.0876355672826225, Theta_dot: -0.15121802640169202
Time: 0.18, Theta: 0.08443844241492314, Theta_dot: -0.16819709378528414
Time: 0.2, Theta: 0.08090802372512097, Theta_dot: -0.1845139269269209
```

ii)




```
Times, Thetas, Theta_dots:
Time: 0.0, Theta: 1, Theta_dot: 0
Time: 0.02, Theta: 0.9983390028529902, Theta_dot: -0.16609971470098026
Time: 0.04, Theta: 0.9933577851791343, Theta_dot: -0.33184421820230925
Time: 0.06, Theta: 0.9850670242717536, Theta_dot: -0.4968733927544581
Time: 0.08, Theta: 0.973484670098038, Theta_dot: -0.6608178092251259
Time: 0.1, Theta: 0.9586361762820105, Theta_dot: -0.823294390153474
Time: 0.12, Theta: 0.940554814602484, Theta_dot: -0.9839024610151095
Time: 0.14, Theta: 0.9192820639188475, Theta_dot: -1.1422203231322636
Time: 0.16, Theta: 0.894868061748177, Theta_dot: -1.297802483330515
Time: 0.18, Theta: 0.8673721040061252, Theta_dot: -1.4501776753650435
Time: 0.2, Theta: 0.8368631757384908, Theta_dot: -1.598847804998203
```

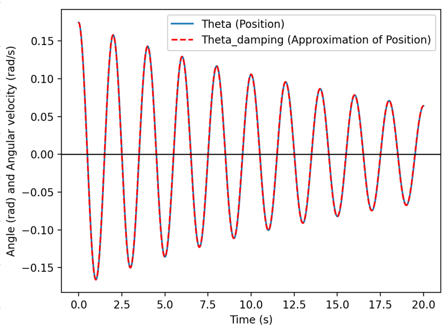
d) By looking at the graphs, we can obviously see that when $\theta_0 = 0.1$, the approximation held. The lines overlap But $\theta_0 = 1$, obviously not. For $\theta_0 = 0.1$, the y-axis of the residuals graph is in units on the scale of $10^{-8} - 10^{-9}$.



2.

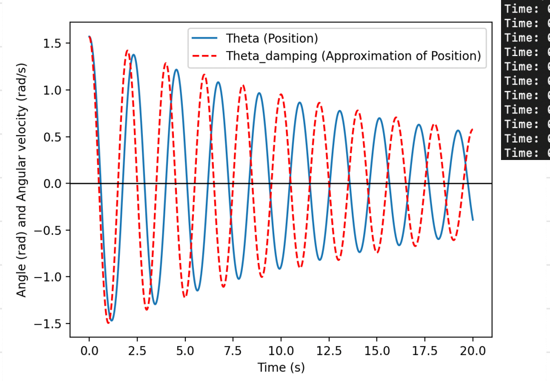
```python
import math
import numpy as np
import matplotlib.pyplot as plt

# initialize array starting at t=0 w/ 1000 steps, dtype=float
N = 1000
times = np.linspace(0, 20, N + 1)
timestep = 0.02

# assign constants
g = 9.8
l = g/(np.pi)**2
b = 0.1 #damping factor
m = 1 #mass of pendulum
gamma = b/m

# initial conditions
theta_0 = np.pi/18    # value from pset (deg converted to radians by *pi/180)
theta_dot_0 = 0    # value from pset

# lists which will be added on to with return values of RK2 function
thetas = [theta_0]
theta_dots = [theta_dot_0]

def RK2(timestep, theta, theta_dot):

    # Filling out everything according to formulas
    k1_theta = timestep * theta_dot
    k1_theta_dot = timestep * ((-gamma * theta_dot) - (g/l * np.sin(theta)))

    k2_theta = timestep * (theta_dot + k1_theta_dot/2)
    k2_theta_dot = timestep * ((-gamma * (theta_dot + k1_theta_dot/2)) - (g/l * np.sin(theta + k1_theta/2)))

    # Update values
    theta_new = theta + k2_theta
    theta_dot_new = theta_dot + k2_theta_dot

    return theta_new, theta_dot_new

theta_current = theta_0
theta_dot_current = theta_dot_0

for i in range(N):
    # assigning return values to current values so the loop can continue
    theta_current, theta_dot_current = RK2(timestep, theta_current, theta_dot_current)
    # adding values to the list
    thetas.append(theta_current)
    theta_dots.append(theta_dot_current)

# more constants
omega_0 = np.sqrt(g/l)
omega = np.sqrt(omega_0**2 - gamma**2/4)

# analytical solutions
theta_no_damping = theta_0 * np.cos(omega * times)
theta_damping = theta_0 * np.exp(-gamma * times/2) * np.cos(omega * times)

# table for pset 2a
print("Times, Thetas, Theta_dots:")
for i in range(11):
    print(f"Time: {times[i]}, Theta: {thetas[i]}, Theta_dot: {theta_dots[i]}")

# plotting
plt.figure()
plt.plot(times, thetas, label="Theta (Position)")
#plt.plot(times, theta_dots, label="Theta_dot (Velocity)", linestyle="-.", color="green")
plt.plot(times, theta_damping, label="Theta_damping (Approximation of Position)", linestyle="--", color="red")
#plt.plot(times, theta_no_damping, label="Theta_no_damping (Approximation with no damping)", linestyle="dotted")
plt.axhline(y=0, color='black', linestyle='-', linewidth=1)
plt.xlabel("Time (s)")
plt.ylabel("Angle (rad) and Angular velocity (rad/s)")
plt.legend()
plt.show()
```

a)
i)



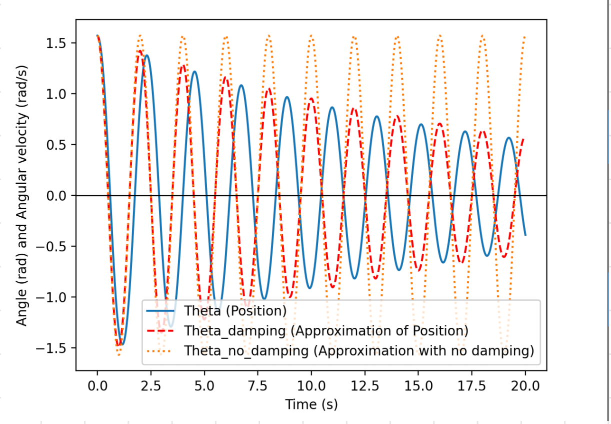

```
Times, Thetas, Theta_dots:
Time: 0.0, Theta: 0.17453292519943295, Theta_dot: 0
Time: 0.02, Theta: 0.1741901574357244, Theta_dot: -0.03424249959448279
Time: 0.04, Theta: 0.17316389086766645, Theta_dot: -0.06828344441930248
Time: 0.06, Theta: 0.17145948151529578, Theta_dot: -0.10199083852166241
Time: 0.08, Theta: 0.16908491297474978, Theta_dot: -0.1352341530505484
Time: 0.1, Theta: 0.16605076243600253, Theta_dot: -0.1678847890713054
Time: 0.12, Theta: 0.16237015747202851, Theta_dot: -0.1998165375051954
Time: 0.14, Theta: 0.15805872365956075, Theta_dot: -0.23090603582780433
Time: 0.16, Theta: 0.15313452310107953, Theta_dot: -0.2610332209227965
Time: 0.18, Theta: 0.14761798393230535, Theta_dot: -0.29008177721297207
Time: 0.2, Theta: 0.14153182091991653, Theta_dot: -0.317939578889854
```

ii)




```
Times, Thetas, Theta_dots:
Time: 0.0, Theta: 1.5707963267948966, Theta_dot: 0
Time: 0.02, Theta: 1.5688224059146787, Theta_dot: -0.19719469593376537
Time: 0.04, Theta: 1.5629085388552597, Theta_dot: -0.3939938605667329
Time: 0.06, Theta: 1.5530626820465139, Theta_dot: -0.5903875559741572
Time: 0.08, Theta: 1.5392931281712454, Theta_dot: -0.7863475465017462
Time: 0.1, Theta: 1.521608962741308, Theta_dot: -0.9818182850159265
Time: 0.12, Theta: 1.5000206998939947, Theta_dot: -1.1767080209079777
Time: 0.14, Theta: 1.474541094564386, Theta_dot: -1.370880084499502
Time: 0.16, Theta: 1.4451861267951664, Theta_dot: -1.5641444365538602
Time: 0.18, Theta: 1.411976151793821, Theta_dot: -1.756249612864936
Time: 0.2, Theta: 1.374937206303802, Theta_dot: -1.9468752411276142
```

b)



← $\theta_0 = 90°$

In both cases, we see a slight discrepancy btw the blue an orange lines, but it's much more pronounced when $\theta_0 = 90°$.

At $\theta_0 = 10°$, the approximation once again almost lined up perfectly.